Please check the examination details below before entering your candidate information

Candidate surname | Other names

Centre Number | Candidate Number

**Pearson Edexcel Level 3 GCE**

# Tuesday 6 June 2023

Afternoon (Time: 2 hours)

Paper reference **9MA0/01**

# Mathematics

## Advanced

## PAPER 1: Pure Mathematics 1

**You must have:**
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over*

P72804A

**1.** Find

$$\int \frac{x^{\frac{1}{2}}(2x-5)}{3}\,\mathrm{d}x$$

writing each term in simplest form.

**(4)**

**(Total for Question 1 is 4 marks)**

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**1.** Find

$$\int \frac{x^{\frac{1}{2}}(3x-2)}{5}\,\mathrm{d}x$$

writing each term in simplest form.

**(4)**

**(Total for Question 1 is 4 marks)**

---

**2.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

$$f(x) = 4x^3 + 5x^2 - 10x + 4a \qquad x \in \mathbb{R}$$

where $a$ is a positive constant.

Given $(x - a)$ is a factor of f($x$),

($a$) show that

$$a(4a^2 + 5a - 6) = 0$$

**(2)**

($b$) Hence

(i) find the value of $a$

(ii) use algebra to find the exact solutions of the equation

$$f(x) = 3$$

**(4)**

**(Total for Question 2 is 6 marks)**

---

**2.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

$$f(x) = 5x^3 + 3x^2 - 7x + 5a \qquad x \in \mathbb{R}$$

where $a$ is a positive constant.

Given $(x - a)$ is a factor of f($x$),

(a) show that

$$a(5a^2 + 3a - 2) = 0$$

**(2)**

(b) Hence

(i) find the value of $a$

(ii) use algebra to find the exact solutions of the equation

$$f(x) = 2$$

**(4)**

**(Total for Question 2 is 6 marks)**

---

**3.** Relative to a fixed origin $O$

- the point $A$ has position vector $5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$
- the point $B$ has position vector $2\mathbf{i} + 4\mathbf{j} + a\mathbf{k}$

where $a$ is a positive integer.

(*a*) Show that $|\overrightarrow{OA}| = \sqrt{38}$

**(1)**

(*b*) Find the smallest value of $a$ for which

$$|\overrightarrow{OB}| > |\overrightarrow{OA}|$$

**(2)**

**(Total for Question 3 is 3 marks)**

---

**3.** Relative to a fixed origin $O$

- the point $A$ has position vector $2\mathbf{i} + 3\mathbf{j} + a\mathbf{k}$
- the point $B$ has position vector $5\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$

where $a$ is a positive integer.

(a) Show that $|\overrightarrow{OB}| = \sqrt{45}$

**(1)**

(b) Find the largest value of $a$ for which

$$|\overrightarrow{OB}| > |\overrightarrow{OA}|$$

**(2)**

**(Total for Question 3 is 3 marks)**

---

**4.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

The curve $C$ has equation $y = \text{f}(x)$ where $x \in \mathbb{R}$

Given that

- $\text{f}'(x) = 2x + \frac{1}{2}\cos x$
- the curve has a stationary point with $x$ coordinate $\alpha$
- $\alpha$ is small

($a$) use the small angle approximation for $\cos x$ to estimate the value of $\alpha$ to 3 decimal places.

**(3)**

The point $P(0, 3)$ lies on $C$

($b$) Find the equation of the tangent to the curve at $P$, giving your answer in the form $y = mx + c$, where $m$ and $c$ are constants to be found.

**(2)**

**(Total for Question 4 is 5 marks)**

---

**4.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

The curve $C$ has equation $y = \text{f}(x)$ where $x \in \mathbb{R}$

Given that

- $\text{f}'(x) = 3x - \frac{1}{3}\cos x$
- the curve has a stationary point with $x$ coordinate $\alpha$
- $\alpha$ is small

(a) use the small angle approximation for $\cos x$ to estimate the value of $\alpha$ to 3 decimal places.

**(3)**

The point $P(0, 5)$ lies on $C$

(b) Find the equation of the tangent to the curve at $P$, giving your answer in the form $y = mx + c$, where $m$ and $c$ are constants to be found.

**(2)**

**(Total for Question 4 is 5 marks)**

---

**5.** A continuous curve has equation $y = f(x)$.

The table shows corresponding values of $x$ and $y$ for this curve, where $a$ and $b$ are constants.

| $x$ | 3 | 3.2 | 3.4 | 3.6 | 3.8 | 4 |
|---|---|---|---|---|---|---|
| $y$ | $a$ | 16.8 | $b$ | 20.2 | 18.7 | 13.5 |

The trapezium rule is used, with all the $y$ values in the table, to find an approximate area under the curve between $x = 3$ and $x = 4$

Given that this area is 17.59

(*a*) show that $a + 2b = 51$

**(3)**

Given also that the sum of all the $y$ values in the table is 97.2

(*b*) find the value of $a$ and the value of $b$

**(3)**

**(Total for Question 5 is 6 marks)**

---

**5.** A continuous curve has equation $y = f(x)$.

The table shows corresponding values of $x$ and $y$ for this curve, where $a$ and $b$ are constants.

| $x$ | 5 | 5.3 | 5.6 | 5.9 | 6.2 | 6.5 |
|---|---|---|---|---|---|---|
| $y$ | 17.6 | $a$ | 21.2 | 20.3 | 17.6 | $b$ |

The trapezium rule is used, with all the $y$ values in the table, to find an approximate area under the curve between $x = 5$ and $x = 6.5$

Given that this area is 28.425

(a) show that $2a + b = 53.7$

**(3)**

Given also that the sum of all the $y$ values in the table is 110.1

(b) find the value of $a$ and the value of $b$

**(3)**

**(Total for Question 5 is 6 marks)**

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**6.** $a = \log_2 x \qquad b = \log_2(x + 8)$

Express in terms of $a$ and/or $b$

(*a*) $\log_2 \sqrt{x}$

**(1)**

(*b*) $\log_2(x^2 + 8x)$

**(2)**

(*c*) $\log_2 \left(8 + \frac{64}{x}\right)$

Give your answer in simplest form.

**(3)**

**(Total for Question 6 is 6 marks)**

---

**6.** $a = \log_2 x \qquad b = \log_2(x + 4)$

Express in terms of $a$ and/or $b$

(a) $\log_2 \frac{1}{\sqrt[3]{x}}$

**(1)**

(b) $\log_2(2x^2 + 8x)$

**(2)**

(c) $\log_2 \left(\frac{16}{x} + 4\right)$

Give your answer in simplest form.

**(3)**

**(Total for Question 6 is 6 marks)**

---

**7.** The function f is defined by

$$f(x) = 3 + \sqrt{x-2} \qquad x \in \mathbb{R} \qquad x > 2$$

(*a*) State the range of f

**(1)**

(*b*) Find $f^{-1}$

**(3)**

The function g is defined by

$$g(x) = \frac{15}{x-3} \qquad x \in \mathbb{R} \qquad x \neq 3$$

(*c*) Find gf(6)

**(2)**

(*d*) Find the exact value of the constant *a* for which

$$f(a^2 + 2) = g(a)$$

**(2)**

**(Total for Question 7 is 8 marks)**

---

**7.** The function f is defined by

$$f(x) = 2 + \sqrt{x-3} \qquad x \in \mathbb{R} \qquad x > 3$$

(a) State the range of f

**(1)**

(b) Find $f^{-1}$

**(3)**

The function g is defined by

$$g(x) = \frac{10}{x-2} \qquad x \in \mathbb{R} \qquad x \neq 2$$

(c) Find gf(4)

**(2)**

(d) Find the exact value of the constant *a* for which

$$f(a^2 + 3) = g(a)$$

**(2)**

**(Total for Question 7 is 8 marks)**

---

**8.**

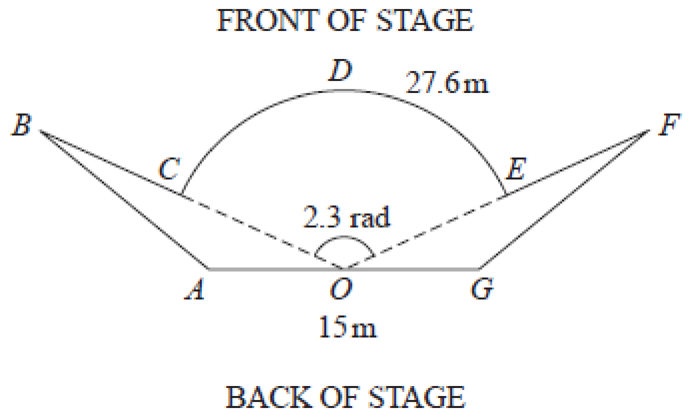


Diagram **NOT** accurately drawn

**Figure 1**

Figure 1 shows the plan view of a stage.

The plan view shows two congruent triangles $ABO$ and $GFO$ joined to a sector $OCDEO$ of a circle, centre $O$, where

- angle $COE = 2.3$ radians
- arc length $CDE = 27.6$ m
- $AOG$ is a straight line of length 15 m

(*a*) Show that $OC = 12$ m.

**(2)**

(*b*) Show that the size of angle $AOB$ is 0.421 radians correct to 3 decimal places.

**(2)**

Given that the total length of the front of the stage, $BCDEF$, is 35 m,

(*c*) find the total area of the stage, giving your answer to the nearest square metre.

**(6)**

**(Total for Question 8 is 10 marks)**

**8.**

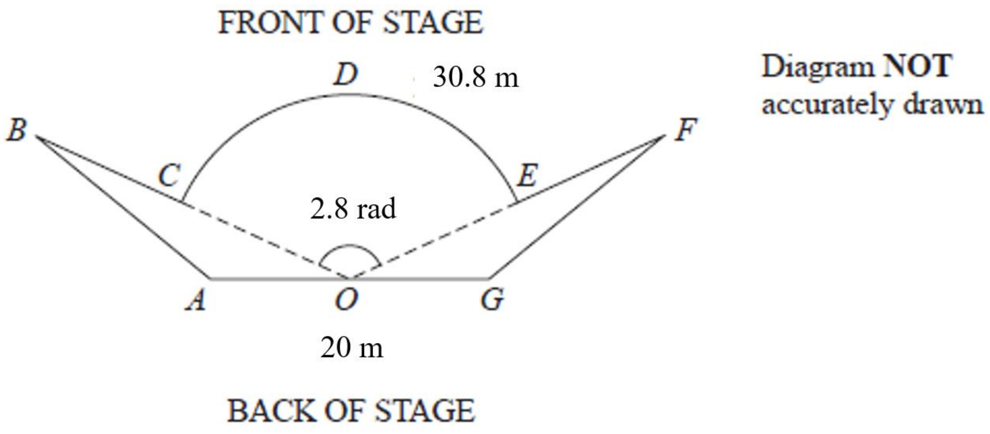


**Figure 1**

Figure 1 shows the plan view of a stage.

The plan view shows two congruent triangles *ABO* and *GFO* joined to a sector *OCDEO* of a circle, centre *O*, where

- angle *COE* = 2.8 radians
- arc length *CDE* = 30.8 m
- *AOG* is a straight line of length 20 m

(a) Show that *OC* = 11 m.

**(2)**

(b) Show that the size of angle *AOB* is 0.171 radians correct to 3 decimal places.

**(2)**

Given that the total length of the front of the stage, *BCDEF*, is 50 m,

(c) find the total area of the stage, giving your answer to the nearest square metre.

**(6)**

**(Total for Question 8 is 10 marks)**

---

**9.** The first three terms of a geometric sequence are

$$3k + 4 \qquad 12 - 3k \qquad k + 16$$

where $k$ is a constant.

(*a*) Show that $k$ satisfies the equation

$$3k^2 - 62k + 40 = 0$$

**(2)**

Given that the sequence converges,

(*b*) (i) find the value of $k$, giving a reason for your answer,

(ii) find the value of $S_\infty$

**(5)**

**(Total for Question 9 is 7 marks)**

---

**9.** The first three terms of a geometric sequence are

$$3k - 5 \qquad 2k - 8 \qquad \frac{5k+1}{5}$$

where $k$ is a constant.

(a) Show that $k$ satisfies the equation

$$5k^2 - 138k + 325 = 0$$

**(2)**

Given that the sequence converges,

(b) (i) find the value of $k$, giving a reason for your answer,

(ii) find the value of $S_\infty$

**(5)**

**(Total for Question 9 is 7 marks)**

---

**10.** A circle $C$ has equation

$$x^2 + y^2 + 6kx - 2ky + 7 = 0$$

where $k$ is a constant.

(*a*) Find in terms of $k$,

(i) the coordinates of the centre of $C$

(ii) the radius of $C$

**(3)**

The line with equation $y = 2x - 1$ intersects $C$ at 2 distinct points.

(*b*) Find the range of possible values of $k$.

**(6)**

**(Total for Question 10 is 9 marks)**

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**10.** A circle $C$ has equation

$$x^2 + y^2 - 2kx + 4ky + 5 = 0$$

where $k$ is a constant.

(a) Find in terms of $k$,

(i) the coordinates of the centre of $C$

(ii) the radius of $C$

**(3)**

The line with equation $y = x - 2$ intersects $C$ at 2 distinct points.

(b) Find the range of possible values of $k$.

**(6)**

**(Total for Question 10 is 9 marks)**

---

**11.**

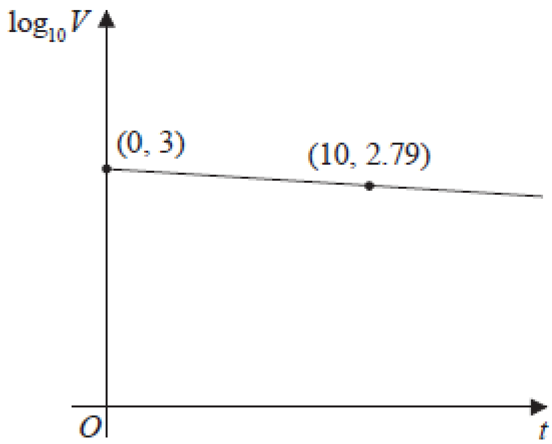


**Figure 2**

The value, $V$ pounds, of a mobile phone, $t$ months after it was bought, is modelled by

$$V = ab^t$$

where $a$ and $b$ are constants.

Figure 2 shows the linear relationship between $\log_{10} V$ and $t$.

The line passes through the points (0, 3) and (10, 2.79) as shown.

Using these points,

(*a*) find the initial value of the phone,

**(2)**

(*b*) find a complete equation for $V$ in terms of $t$, giving the exact value of $a$ and giving the value of $b$ to 3 significant figures.

**(3)**

Exactly 2 years after it was bought, the value of the phone was £320

(*c*) Use this information to evaluate the reliability of the model.

**(2)**

**(Total for Question 11 is 7 marks)**

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**11.**

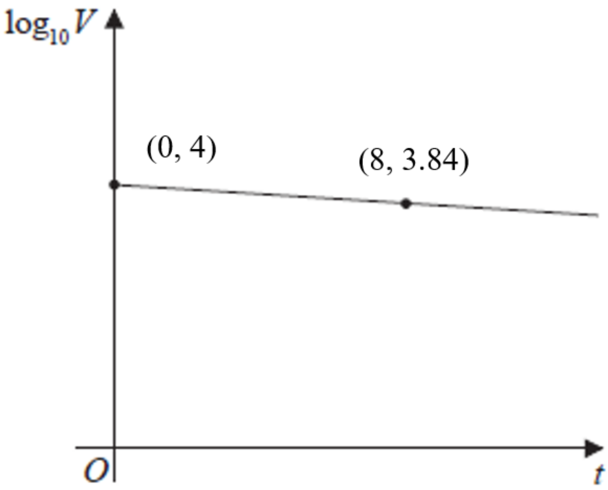


**Figure 2**

The value, $V$ pounds, of a car, $t$ months after it was bought, is modelled by

$$V = ab^t$$

where $a$ and $b$ are constants.

Figure 2 shows the linear relationship between $\log_{10} V$ and $t$.

The line passes through the points (0, 4) and (8, 3.84) as shown.

Using these points,

(a) find the initial value of the car.

**(2)**

(b) find a complete equation for $V$ in terms of $t$, giving the exact value of $a$ and the value of $b$ to 3 significant figures.

**(3)**

Exactly 2 years after it was bought, the value of the car was £3300

(c) Use this information to evaluate the reliability of the model.

**(2)**

**(Total for Question 11 is 7 marks)**

**12.** $$y = \sin x$$

where $x$ is measured in radians.

Use differentiation from first principles to show that

$$\frac{dy}{dx} = \cos x$$

You may

- use without proof the formula for $\sin(A \pm B)$
- assume that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

**(5)**

**(Total for Question 12 is 5 marks)**

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**12.** $$y = \cos x$$

where $x$ is measured in radians.

Use differentiation from first principles to show that

$$\frac{dy}{dx} = -\sin x$$

You may

- use without proof the formula for $\cos(A \pm B)$
- assume that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

**(5)**

**(Total for Question 12 is 5 marks)**

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**13.** On a roller coaster ride, passengers travel in carriages around a track.

On the ride, carriages complete multiple circuits of the track such that

- the maximum vertical height of a carriage above the ground is 60 m
- a carriage starts a circuit at a vertical height of 2 m above the ground
- the ground is horizontal

The vertical height, $H$ m, of a carriage above the ground, $t$ seconds after the carriage starts the first circuit, is modelled by the equation

$$H = a - b(t - 20)^2$$

where $a$ and $b$ are positive constants.

(*a*) Find a complete equation for the model.

**(3)**

(*b*) Use the model to determine the height of the carriage above the ground when $t = 40$

**(1)**

In an alternative model, the vertical height, $H$ m, of a carriage above the ground, $t$ seconds after the carriage starts the first circuit, is given by

$$H = 29\cos(9t + \alpha)^\circ + \beta \qquad 0 \leq \alpha < 360^\circ$$

where $\alpha$ and $\beta$ are constants.

(*c*) Find a complete equation for the alternative model.

**(2)**

Given that the carriage moves continuously for 2 minutes,

(*d*) give a reason why the alternative model would be more appropriate.

**(1)**

**(Total for Question 13 is 7 marks)**

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**13.** On a roller coaster ride, passengers travel in carriages around a track.

On the ride, carriages complete multiple circuits of the track such that

- the maximum vertical height of a carriage above the ground is 42 m
- a carriage starts a circuit at a vertical height of 1.5 m above the ground
- the ground is horizontal

The vertical height, $H$ m, of a carriage above the ground, $t$ seconds after the carriage starts the first circuit, is modelled by the equation

$$H = a - b(t - 15)^2$$

where $a$ and $b$ are positive constants.

(a) Find a complete equation for the model.

**(3)**

(b) Use the model to determine the height of the carriage above the ground when $t = 25$

**(1)**

In an alternative model, the vertical height, $H$ m, of a carriage above the ground, $t$ seconds after the carriage starts the first circuit, is given by

$$H = 20.25\cos(12t + \alpha)° + \beta \qquad 0 \leqslant \alpha < 360°$$

where $\alpha$ and $\beta$ are constants.

(c) Find a complete equation for the alternative model.

**(2)**

Given that the carriage moves continuously for 2 minutes,

(d) give a reason why the alternative model would be more appropriate.

**(1)**

**(Total for Question 13 is 7 marks)**

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**14.** Prove, using algebra, that

$$(n + 1)^3 - n^3$$

is odd for all $n \in \mathbb{N}$

**(4)**

**(Total for Question 14 is 4 marks)**

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**14.** Prove, using algebra, that

$$(n - 1)^3 - n^3$$

is odd for all $n \in \mathbb{N}$

**(4)**

**(Total for Question 14 is 4 marks)**

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**15.** A curve has equation $y = \text{f}(x)$, where

$$\text{f}(x) = \frac{7xe^x}{\sqrt{e^{3x} - 2}} \qquad x > \ln \sqrt[3]{2}$$

(*a*) Show that

$$\text{f}'(x) = \frac{7e^x\left(e^{3x}(2-x) + Ax + B\right)}{2(e^{3x}-2)^{\frac{3}{2}}}$$

where $A$ and $B$ are constants to be found.

**(5)**

(*b*) Hence show that the $x$ coordinates of the turning points of the curve are solutions of the equation

$$x = \frac{2e^{3x} - 4}{e^{3x} + 4}$$

**(2)**

The equation $x = \dfrac{2e^{3x} - 4}{e^{3x} + 4}$ has two positive roots $\alpha$ and $\beta$ where $\beta > \alpha$

A student uses the iteration formula

$$x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$$

in an attempt to find approximations for $\alpha$ and $\beta$

Diagram 1 shows a plot of part of the curve with equation $y = \dfrac{2e^{3x} - 4}{e^{3x} + 4}$ and part of the line with equation $y = x$

Using Diagram 1 below

(*c*) draw a staircase diagram to show that the iteration formula starting with $x_1 = 1$ can be used to find an approximation for $\beta$

**(1)**

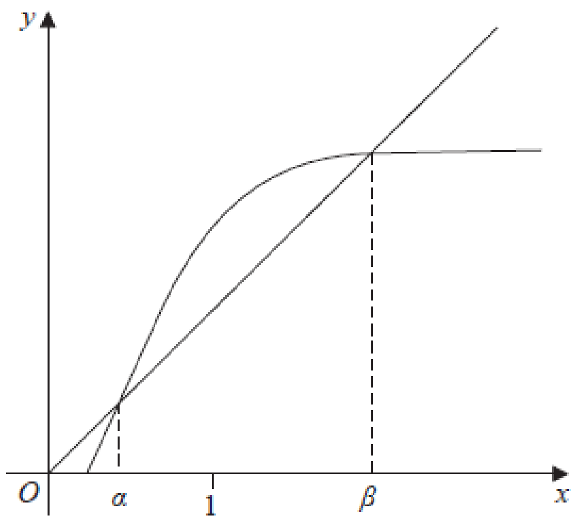


**Diagram 1**

Use the iteration formula with $x_1 = 1$, to find, to 3 decimal places,

(*d*) (i) the value of $x_2$

(ii) the value of $\beta$

**(3)**

Using a suitable interval and a suitable function that should be stated

(*e*) show that $\alpha = 0.432$ to 3 decimal places.

**(2)**

**(Total for Question 15 is 13 marks)**

**15.** A curve has equation $y = f(x)$, where

$$f(x) = \frac{5xe^x}{\sqrt{e^{3x} - 3}} \qquad x > \ln \sqrt[3]{3}$$

(*a*) Show that

$$f'(x) = \frac{5e^x(e^{3x}(2 - x) + Ax + B)}{2(e^{3x} - 3)^{\frac{3}{2}}}$$

where *A* and *B* are constants to be found.

**(5)**

(*b*) Hence show that the *x* coordinates of the turning points of the curve are solutions of the equation

$$x = \frac{2e^{3x} - 6}{e^{3x} + 6}$$

**(2)**

The equation $x = \frac{2e^{3x} - 6}{e^{3x} + 6}$ has two positive roots $\alpha$ and $\beta$ where $\beta > \alpha$

A student uses the iteration formula

$$x_{n+1} = \frac{2e^{3x_n} - 6}{e^{3x_n} + 6}$$

in an attempt to find approximations for $\alpha$ and $\beta$

Diagram 1 shows a plot of part of the curve with equation $y = \frac{2e^{3x} - 6}{e^{3x} + 6}$ and part of the line with equation $y = x$

Using Diagram 1 below

(*c*) draw a staircase diagram to show that the iteration formula starting with $x_1 = 1$ can be used to find an approximation for $\beta$

**(1)**

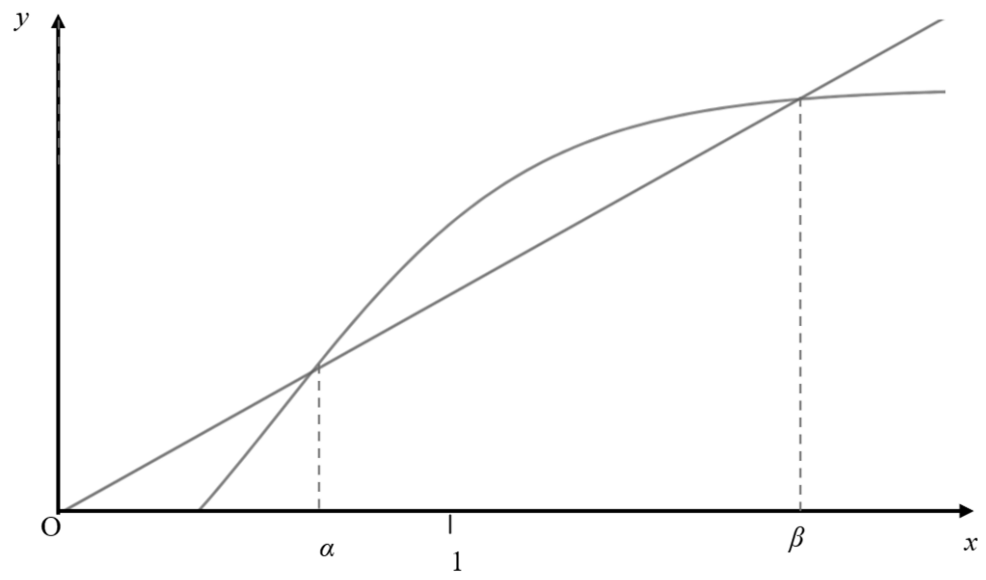


**Diagram 1**

Use the iteration formula with $x_1 = 1$, to find, to 3 decimal places,

($d$) (i) the value of $x_2$

(ii) the value of $\beta$

**(3)**

Using a suitable interval and a suitable function that should be stated

($e$) show that $\alpha = 0.676$ to 3 decimal places.

**(2)**

**(Total for Question 15 is 13 marks)**